

# A Study of Gluon Propagator on Coarse Lattice

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## Abstract

We study gluon propagator in Landau gauge with lattice QCD, where we use an improved lattice action. The calculation of gluon propagator is performed on lattices with the lattice spacing from 0.40fm to 0.24fm and with the lattice volume from  $(2.40\text{fm})^4$  to  $(4.0\text{fm})^4$ . We try to fit our results by two different ways, in the first one we interpret the calculated gluon propagators as a function of the continuum momentum, while in the second we interpret the propagators as a function of the lattice momentum. In the both we use models which are the same in continuum limit. A qualitative agreement between two fittings is found.

PACS numbers: 11.15Ha, 12.38.Gc, 12.38.Aw, 14.70.Dj

## 1. Introduction

Lattice QCD enables us to study the nonperturbative nature of QCD from the first principle. As space-time is replaced with a discreted lattice, physical results from Monte-Carlo simulations can be obtained if the effect of the finite lattice spacing  $a$  is under control. For this purpose simulations of lattice QCD are performed on large and fine lattices to ensure that the effect is small enough. This is expensive. Using an improved action to simulate lattice QCD on small and coarse lattices, it is possible to obtain physical results with less expense. The latter was proposed long time ago [1]. It was suggested that one can use perturbative theory to improve a lattice action and the effect of the finite lattice spacing  $a$  can systematically be removed. However, for lattice QCD perturbative series converge so slowly that the improvement is not significant. It was pointed out recently that perturbative theory can be made more effective by employing the tadpole improvement [2]. With an improved action implemented with the tadpole improvement for lattice QCD some physical results are already obtained from simulations on a coarse lattice. For example, hadron masses are well determined [3–6].

In this paper we study gluon propagator in Landau gauge with an improved action on coarse lattices, where quarks are neglected. Perturbatively a gluon is expected to behave like a massless particle. Because of the color confinement of QCD such expectation can not be correct in a real world. A nonperturbative study of gluon propagator is required and can help us to understand the color confinement. Previous studies [12–16] are undertaken with the Wilson action, i.e., an unimproved action, they already show that gluon propagator is more complicated than that from perturbative theory. Especially, in Landau gauge it is well known that gluon propagator is infrared finite due to Gribov’s copies [10], this is also proven to be true on lattice [11].

We study gluon propagator on a series of lattices, of which the lattice spacing  $a$  is from 0.40fm and to 0.22fm. At  $a = 0.40\text{fm}$  we also investigate gluon propagator with lattice volume from  $(2.40\text{fm})^4$  to  $(4.0\text{fm})^4$ . This allows us to study the effect of the finite lattice

spacing and of the finite volume. The whole calculation is performed with a PC running at 400MHz.

In continuum the gluon propagator  $D_{\mu\nu}^{ab}$  in Landau gauge is characterized by a single function  $D(q^2)$ :

$$D_{\mu\nu}^{ab}(q) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) \delta_{ab} D(q^2). \quad (1)$$

The index  $a$  and  $b$  is the color index. We define a lattice version of the function  $D(q^2)$ , which is denoted as  $D_L$ , and calculate it. We find that the function  $D_L$  can be well interpreted as a function of  $q^2$  for  $q^2$  up to  $3.5(\text{GeV})^2$ . We fit our results with a model proposed in [12]. As our lattice spacing is rather larger, the effect of the finite lattice spacing needs to be studied in detail. For this we also try to fit our propagator as a function of a modified lattice momentum variable, where the same model is used with a slight modification suggested by perturbative theory. An qualitative agreement is found between these two fittings.

Our paper is organized as the following: In Sect. 2 we introduce the action and our notation, we discuss different interpretations of our data. In Sect.3 we fit our results by taking  $D_L$  as a function of  $q^2$  with the model in [12]. In Sect.4 we fit our results by taking  $D_L$  as a function of lattice momentum with the same model, but with a slight modification as suggested by perturbative theory. In the both sections we will discuss the effect of the finite lattice spacing and of the finite volume on our fitting results. Sect.5 is our conclusion.

## 2. The Action and Our Notations

We take the one-loop improved action for gluon [7], where the action consists of plaquette, rectangle and parallelogram terms and is accurate up to errors of  $O(\alpha_s^2 a^2, a^4)$ . Implementing tadpole improvement the action becomes [3]

$$\begin{aligned} S(U) &= \beta \sum_{pl} \frac{1}{3} \text{ReTr}(1 - U_{pl}) + \beta_{rt} \sum_{rt} \frac{1}{3} \text{ReTr}(1 - U_{tr}) + \beta_{pg} \sum_{pg} \frac{1}{3} \text{ReTr}(1 - U_{pg}), \\ \beta_{rt} &= -\frac{\beta}{20u_0^2} (1 + 0.4805\alpha_s), \quad \beta_{pg} = -\frac{\beta}{u_0^2} 0.03325\alpha_s, \\ u_0 &= (\frac{1}{3} \text{ReTr}\langle U_{pl} \rangle)^{\frac{1}{4}}, \quad \alpha_s = -\frac{\ln(\frac{1}{3} \text{ReTr}\langle U_{pl} \rangle)}{3.06839} \end{aligned} \quad (2)$$

We used this action to generate gluonic configurations at  $\beta = 6.8$ ,  $\beta = 7.1$  and  $\beta = 7.4$ , where the lattice size is  $6^4$ ,  $8^4$  and  $10^4$  respectively. The parameter  $u_0$  is determined by self-consistency. We also generate configurations at  $\beta = 6.8$  with lattice size of  $8^4$  and  $10^4$ . The configurations are generated with the pseudo heat bath method [8], where the three  $SU(3)$  subgroups were updated 3 times in each overall update step. Configurations are separated by 40 sweeps to ensure that they are statistically independent. The configurations used in this work are summarized in Table 1.

**Table 1**

$\beta$	$L^4$	No.	$a$	$u_0$
6.8	$6^4$	500	0.40fm	0.8267
6.8	$8^4$	200	0.40fm	0.8267
6.8	$10^4$	200	0.40fm	0.8267
7.1	$8^4$	500	0.33fm	0.8434
7.4	$10^4$	250	0.24fm	0.8631

In Table 1 the lattice spacing  $a$  is determined by the string tension [3]. We use the naive steepest descent method to fix the gauge of our configurations [9]. We define:

$$\begin{aligned}\Delta(x) &= \sum_{\nu} [U_{\nu}(x - a\nu) - U_{\nu}(x) - h.c - \text{trace term}], \\ \theta &= \frac{1}{3V} \sum_x \text{Tr}(\Delta(x)\Delta^{\dagger}(x)),\end{aligned}\tag{3}$$

where  $V = L^4$  is the lattice volume. By gauge transformation we try to minimize the quantity  $\theta$ .  $\theta = 0$  corresponds to the Landau gauge. Numerically we require  $\theta < 10^{-8}$ . The gauge field is defined as

$$A_{\mu}(x + a\hat{\mu}/2) = \frac{1}{2ia g_0}(U_{\mu}(x) - U_{\mu}^{\dagger}(x)) - \frac{1}{6ia g_0}\text{Tr}(U_{\mu}(x) - U_{\mu}^{\dagger}(x)).\tag{4}$$

It should be noted that in the Wilson action the bar coupling  $g_0$  is related to the  $\beta$  as  $g_0\beta = 6$ . With the action in Eq.(2) the bar coupling  $g_0$  is related to  $\beta$  as

$$g_0^2 = \frac{10}{\beta}, \quad (5)$$

if one neglects the tadpole improvement and the correction in the action at one-loop level. Adding the improvement and the correction the relation becomes complicated. In Landau gauge the quantity  $\sum_{\mathbf{x}} A_0(t, \mathbf{x})$  is independent on  $t$  for each configuration. We checked this under the numerical condition for  $\theta$ . The quantity varies at different  $t$  under 0.1%.

The Fourier transformed gauge field is

$$A_\mu(q) = a^4 \sum_x e^{-iq \cdot (x + a\hat{\mu}/2)} A_\mu(x), \quad (6)$$

where  $q$  is the momentum:

$$q_\mu = \frac{2\pi}{L} n_\mu, \quad n_\mu = 0, 1, \dots, L-1. \quad (7)$$

We define a function on lattice as

$$D_L(q) = \frac{g_0^2}{12V} \sum_\mu \langle 0 | \text{Tr}(A_\mu(q) A_\mu^\dagger(q)) | 0 \rangle. \quad (8)$$

In the limit of  $a \rightarrow 0$  the function  $D_L(q)$  will approach to the function  $D(q^2)$  in the sense that it will be proportional to  $D(q^2)$ . We will calculate the function  $D_L(q)$  with the configurations listed in Table 1. For further convenience we introduce

$$\hat{q}_\mu = \frac{2}{a} \sin\left(\frac{1}{2} a q_\mu\right), \quad \hat{q}^2 = \sum_\mu \hat{q}_\mu^2, \quad \hat{q}^4 = \sum_\mu \hat{q}_\mu^4. \quad (9)$$

In general the calculated function  $D_L(q)$  will depend on  $\hat{q}^2$ ,  $\hat{q}^4$  and other possible invariants on lattice. if  $a q_\mu$  is small enough,  $D_L(q)$  will approximately depend only on the variable

$$q^2 = \sum_\mu q_\mu^2. \quad (10)$$

Previous calculation on large lattice shows that even for  $a^2 q^2 < 1$   $D_L(q)$  still can not be interpreted as a function of  $q^2$ , the data points for  $D_L(q)$  with momenta directed along one of the four axes behave differently than those with momenta directed off axis and the

difference is significant. This clearly indicates that the rotation invariance is violated due to finite lattice spacing and the effect is large. With a improved action like the one in Eq.(2) the effect of the violation will be reduced. Our data for  $D_L(q)$  from the configurations with  $\beta = 7.4$  is shown in Fig.1a, where  $D_L(q)$  is plotted as a function of  $q^2$ , and data with same  $q^2$  are averaged. From the figure we see that the data points behave well as a function of  $q^2$  in a range from  $a^2q^2 = 0$  to  $a^2q^2 = 5.13$ . For  $a^2q^2 > 5.13$  the effect due to the violation of the rotation invariance becomes significant, indicated by a small jump in our data around  $a^2q^2 = 6.3$ . This is clearly shown by the data from our smallest lattice in Fig.1b, where a small jump is roughly at  $a^2q^2 = 10.0$  or at  $q^2 = 2.5\text{GeV}^2$ , for  $a^2q^2 > 8.0$  the data points become unregular. The similar behave is also found for the lattice with the size  $L = 8$ . We plot our data from the lattice with  $\beta = 7.1$  in Fig.1c. We will try to fit our data lying between  $q^2 = 0$  and the  $q^2$ , where the data points begin to become unregular, by taking  $D_L$  as a function of  $q^2$ .

However it may be wondered that the effect of the finite lattice spacing is small in the above consideration. To study this we also try to fit our results by taking  $D_L(q)$  as a function of  $\hat{q}^2$ . The replacement of  $q^2$  by  $\hat{q}^2$  can be thought as kinematic correction. In Fig.2a we plot  $D_L(q)$  from the lattice with  $\beta = 7.4$  as a function of  $\hat{q}^2$ , where data points with adjacent momenta lying within  $\delta a^2\hat{q}^2 < 0.0001$  are averaged. From Fig.2a one can see that the data is not well scaled by  $\hat{q}^2$  and this indicates that  $D_L$  may depends on on other lattice invariants and the kinematic correction needs to be modified. Perturbative calculation with the action in Eq.(2) also shows that  $D_L(q)$  does not only depend on  $\hat{q}^2$  but also on  $\hat{q}^4$ . At the tree-level without the tadpole improvement one can obtain [17]

$$D_L(q) = \frac{g_0^2}{3} \left\{ -\frac{a^2}{12} + 3 \left[ \frac{1}{\hat{q}^2} - \frac{a^2}{18} \cdot \frac{\hat{q}^4}{(\hat{q}^2)^2} \right] \right\} + O(a^4) \quad (11)$$

where there is a constant term. With this in mind the kinematic correction above needs to be further modified. We introduce a new variable:

$$\hat{q}_L^2 = \hat{q}^2 + \frac{a^2}{12} \hat{q}^4, \quad (12)$$

and for  $a \rightarrow 0$ :

$$\hat{q}_L^2 = q^2 + O(a^4). \quad (13)$$

In Fig. 2b we plot the gluon propagator as a function of  $\hat{q}_L^2$ , where data points with adjacent momenta lying within  $\delta a^2 \hat{q}_L^2 < 0.0001$  are averaged. From the figure one can see that  $D_L$  can be interpreted as a function of  $\hat{q}_L^2$ , although there is still a small fluctuation among the data points, but it is much better than interpreting  $D_L$  as a function of  $\hat{q}^2$ . We will also fit our results by taking  $D_L$  as a function of  $\hat{q}_L^2$ .

It should be pointed out that only on-shell quantities calculated with an improved action are improved. For an operator in general one needs to improve it to remove effect of finite  $a$ . In our case it is complicated to do so. The reason is that the primary field on lattice is the gauge link  $U_\mu(x)$ , certain effect of the finite  $a$  at order of  $a^2$  is introduced by extracting the gauge field  $A_\mu(x)$  from  $U_\mu(x)$ , and some effect at order of  $a^2$  arises if one calculate the propagator. To obtain an improved  $D_L(q)$  one should improve the extraction and then improve the operator  $A_\mu(q)A_\mu^\dagger(q)$ . In this work we will not consider to obtain an improved  $D_L(q)$ .

### 3. Fitting propagators as a function of continuum momentum

In this section we fit our calculated propagators as functions of  $q^2$  with the model [12]

$$D_L(q^2) = \frac{Z}{(M^2)^{1+\alpha} + (q^2)^{1+\alpha}} \quad (14)$$

where  $M$  is a dimensional parameter and  $\alpha$  is the anomalous dimension.  $Z$  is a parameter related to  $g_0$  and the renormalization constant of wave-function. In this and the next section we will not discuss this parameter. In the fitting of our propagators we exclude the data points with the lowest  $q^2$  including the point with  $q^2 = 0$ . We try to include more data points with larger  $q^2$  until  $\chi^2$  per degree of freedom close to 1, but not larger than 1. Beside this the total fitting range is limited because of the reason discussed in the last section. Our fitting results are summarized in Table 2:

**Table 2**

$\beta$	$L$	Range	$Z$	$a^2 M^2$	$\alpha$	$\chi^2/dof$
6.8	8	3-9	5.811(133)	1.654(92)	0.107(24)	0.35
6.8	10	3-12	5.975(77)	1.730(49)	0.111(14)	0.07
7.1	8	3-12	5.511(14)	0.949(106)	0.187(18)	0.28
7.4	10	3-14	5.298(75)	0.602(41)	0.312(13)	0.08

The data points are labeled by an integer  $n$ , smaller  $n$  corresponds to smaller  $q^2$ ,  $n = 1$  corresponds to  $q^2 = 0$ . The range is the fitting range and given by this integer. We do not list our results for the configurations with  $\beta = 6.8, L = 6$ , because the fitting quality is poor with small number of data points. We also tried to fit our data with other models. For example, the model proposed in [15], but it can not well describe our data in the range given above by the indication of large  $\chi^2/dof$ . To illustrate our fitting quality we plot our data with fitting results in Fig.3a and in Fig.3b for  $\beta = 7.4$  and for  $\beta = 6.8$  respectively.

From our fitting results and with the lattice spacing given in Table 1 we obtain the dimensional parameter  $M$  in GeV:

$$\begin{aligned}
M &= 0.633(28) \text{ for } \beta = 6.8, L = 8, \quad M = 0.647(14) \text{ for } \beta = 6.8, L = 10, \\
M &= 0.582(56) \text{ for } \beta = 7.1, L = 8, \quad M = 0.636(34) \text{ for } \beta = 7.4, L = 10.
\end{aligned} \tag{15}$$

We note that the values of  $M$  at  $\beta = 6.8$  with  $L = 8$  and with  $L = 10$  can be taken as the same within the errors, this may indicate that the effect of the finite volume is small. However, we have only two values from two lattices, where the difference in volume is not large, a study on larger lattices with different volumes is needed to confirm the above conclusion. From above the values of  $M$  remain relatively stable with different lattice spacings  $a$ . We use a standard weighted least-squares procedure to average the  $M$ - values and obtain the central value  $M_c$ :

$$M_c = 641(11)\text{MeV}. \tag{16}$$



We also note that the deviation of  $M$  at  $\beta = 7.1$  from  $M_c$  is the largest, but it is still in the range of the errors. The deviation of other values is small. This leads to conclude that there is no significant dependence of  $M$  on the finite  $a$  and to interpret the central value  $M_c$  as the parameter  $M$  in the continuum limit.

As the next we study the anomalous dimension  $\alpha$ . From the fitting results in Table. 2, the values from two lattices with different volumes at  $\beta = 6.8$  are the same within the errors. This may also indicate that the effect of the finite volume in  $\alpha$  is small as that in  $M$ . For values at different  $\beta$  there is clearly a  $a$ -dependence of  $\alpha$ . We try to model the dependence with a simple formula:

$$\alpha(a) = \alpha_0 + \alpha_1 a^2 \quad (17)$$

where  $a$  is in unit of fm and  $\alpha_0$  can be thought as the anomalous dimension in continuum. At  $\beta = 6.8$  we take the value of  $\alpha$  with  $L = 10$  for the fit. The results is

$$\alpha_0 = 0.421(20), \quad \alpha_1 = -1.97(19), \quad \chi^2 = 1.42. \quad (18)$$

In Fig. 4 we illustrate the quality of this fit. The value of  $\chi^2$  is acceptable. With the above discussion we conclude in this section with our data that the gluon propagator in continuum has a mass scale at 641(11)MeV and there is an anomalous dimension which is 0.421(20).

#### 4. Fitting propagator as a function of lattice momentum

In this section we fit our data with the same model as in the last section, where the variable  $q^2$  is replaced by  $\hat{q}_L^2$ , and a constant term is added as suggested by perturbative theory. This term will vanish in the limit of  $a \rightarrow 0$ . The model with such modification is:

$$D_L(q^2) = \frac{Z}{(M^2)^{1+\alpha} + (\hat{q}_L^2)^{1+\alpha}} + ca^2. \quad (19)$$

The data selection for fitting is similar as in the last section. We discarded the first two points and try to include more data points in the direction of increasing  $\hat{q}_L^2$ , until  $\chi^2$  per degree of freedom close to 1 but never larger than 1. Our fitting results are summarized in Table 3.

**Table 3**

$\beta$	$L$	Range	$Z$	$a^2 M^2$	$\alpha$	$c$	$\chi^2/dof$
6.8	6	3-12	6.297(72)	1.433(67)	-0.019(8)	-0.319(11)	0.65
6.8	8	3-13	6.204(68)	1.496(56)	-0.080(10)	-0.379(14)	0.50
6.8	10	3-16	6.007(59)	1.440(46)	-0.097(11)	-0.365(14)	0.44
7.1	8	3-13	6.203(69)	0.945(39)	0.026(12)	-0.383(15)	0.76
7.4	10	3-15	5.809(56)	0.546(20)	0.147(13)	-0.361(16)	0.58

The data points are labeled by an integer  $n$ , smaller  $n$  corresponds to smaller  $q_L^2$ ,  $n = 1$  corresponds to  $\hat{q}_L^2 = 0$ . The range is the fitting range and is given by this integer. To illustrate out fitting we plot our data with the fitted curves in Fig.5a and Fig.5b as examples. From above one can realize that the  $c$  roughly remain as the same at different  $\beta$  and at different lattice volumes except the one at  $\beta = 6.8, L = 6$ . For other values of  $c$  the variation is in the range within errors. This fact will ensure that the term with  $c$  in Eq.(19) will vanish in the limit of  $a \rightarrow 0$ .

With the lattice spacing in Table 1 we obtain the mass parameter:

$$\begin{aligned}
M &= 0.590(11) \text{ for } \beta = 6.8, L = 6, \\
M &= 0.602(9) \text{ for } \beta = 6.8, L = 8, \quad M = 0.591(8) \text{ for } \beta = 6.8, L = 10, \\
M &= 0.580(12) \text{ for } \beta = 7.1, L = 8, \quad M = 0.606(42) \text{ for } \beta = 7.4, L = 10.
\end{aligned} \tag{20}$$

We observe that the value of  $M$  is within the errors stable in the range of  $\beta$  and in the range of the volume of our lattices. This also indicates as in the last section that the gluon propagator in continuum limit contains a mass scale. We calculate with the data the central value:

$$M_c = 592(5)\text{MeV}. \tag{21}$$

This value is 10% smaller than the one in the last section. The reason may be due to the

difference between the variables  $q^2$  and  $q_L^2$ , i.e., the effect of the finite lattice spacing is still observable, it may be at order of 10%.

At  $\beta = 6.8$  the parameter  $\alpha$  from the lattice with the volume  $L = 6$  is quite different than the values from other larger lattices, while there is no large deviation between the values from  $L = 8$  and from  $L = 10$ . The similar is also found with the parameter  $c$ . This may be explained that at  $\beta = 6.8$  the effect of the finite volume is significant for  $L$  smaller than 8. We try to fit this dependence with the same relation as in the last section:

$$\alpha = \alpha_0 + \alpha_1 a^2, \quad (22)$$

the fitting results are:

$$\alpha_0 = 0.285(20), \quad \alpha_1 = -2.38(17), \quad \chi^2 = 0.07. \quad (23)$$

The fitting quality is better than that in the last section. But  $\alpha_0$  is different than this in the last section. Here  $\alpha_0$  may not be interpreted as the anomalous dimension. The reason is: In the model in Eq.(14) the effect of the finite  $a$  is only contained in the parameters, while in the model in Eq.(19) not only the parameters implicitly contain possible effect of the finite  $a$ , but also some effect of the finite  $a$  is explicitly included, representing by the term with  $c$  and with the variable  $\hat{q}_L^2$ . If the latter can be neglected, one may interpret  $\alpha_0$  as the anomalous dimension, which is 30% smaller than  $\alpha_0$  in the last section. This may be thought as a qualitative agreement.

## 5. Conclusion

In this work we studied gluon propagator with an improved action on lattice. Unlike with the Wilson action, we find that the effect due to the violation of the rotation invariance is reduced, resulting in that  $D_L$  can be well interpreted as a function of  $q^2$  as a continuum variable for  $a^2 q^2$  not larger than 5.13. We fitted the gluon propagator with a model proposed in [12] and studied the effect of the finite lattice spacing and of the finite volume. Our conclusion is that in the continuum limit the gluon propagator has a mass scale around

600MeV and a anomalous dimension which is 0.412. We also tried to study the gluon propagators as a function of an improved lattice momentum  $q_L^2$  and fitted them with the model which is the same in the continuum limit as the one used before. The fitting results are qualitatively in agreement with the study of  $D_L$  as a function of  $q^2$ . The difference between two fittings may be explained by the effect of the finite lattice spacing, and by the effect depending on how to model the gluon propagator  $D_L$ . Simulations with large lattices may be required to study the difference in detail.

Finally it should be pointed that calculations with larger lattice than those in our work will allow us not only to study the effect of the finite lattice spacing and of the finite volume in more detail but also to have more data points with small  $q^2$ .

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## Figure Caption

Fig.1a: The gluon propagator  $D_L$  at  $\beta = 7.4$  with  $L = 10$  plotted as a function of  $q^2$ . Statistical errors are not plotted, they are small and around 1%.

Fig.1b: The gluon propagator  $D_L$  at  $\beta = 6.8$  with  $L = 6$  plotted as a function of  $q^2$ .

Fig.1c: The gluon propagator  $D_L$  at  $\beta = 7.1$  with  $L = 8$  plotted as a function of  $q^2$ .

Fig.2a: The gluon propagator  $D_L$  at  $\beta = 7.4$  with  $L = 8$  plotted as a function of  $\hat{q}^2$ . The x-axis is for  $a^2\hat{q}^2$ .

Fig.2b: The gluon propagator  $D_L$  at  $\beta = 7.4$  with  $L = 8$  plotted as a function of  $\hat{q}_L^2$ . The x-axis is for  $a^2\hat{q}_L^2$ .

Fig.3a: The gluon propagator  $D_L$  at  $\beta = 7.4$  with  $L = 10$  plotted as a function of  $q^2$ , the line is from our fit. All data shown in this figure are used in fitting.

Fig.3b: The gluon propagator  $D_L$  at  $\beta = 6.8$  with  $L = 8$  plotted as a function of  $q^2$ , the line is from our fit. All data shown in this figure are used in fitting.

Fig.4 The extrapolation of  $\alpha$ . The x-axis is for  $a^2$  in  $(\text{fm})^2$ , the y-axis is for  $\alpha$ .

Fig.5a: The gluon propagator  $D_L$  at  $\beta = 7.4$  with  $L = 10$  plotted as a function of  $\hat{q}_L^2$ , the line is from our fit with the model in Eq.(19). All data shown in this figure are used in fitting.

Fig.5b: The gluon propagator  $D_L$  at  $\beta = 6.8$  with  $L = 6$  plotted as a function of  $\hat{q}_L^2$ , the line is from our fit with the model in Eq.(19). All data shown in this figure are used in fitting.

Fig.6 The extrapolation of  $\alpha$ . The x-axis is for  $a^2$  in  $(\text{fm})^2$ , the y-axis is for  $\alpha$ . The  $\alpha$  is defined in Eq.(19).

Fig.1a

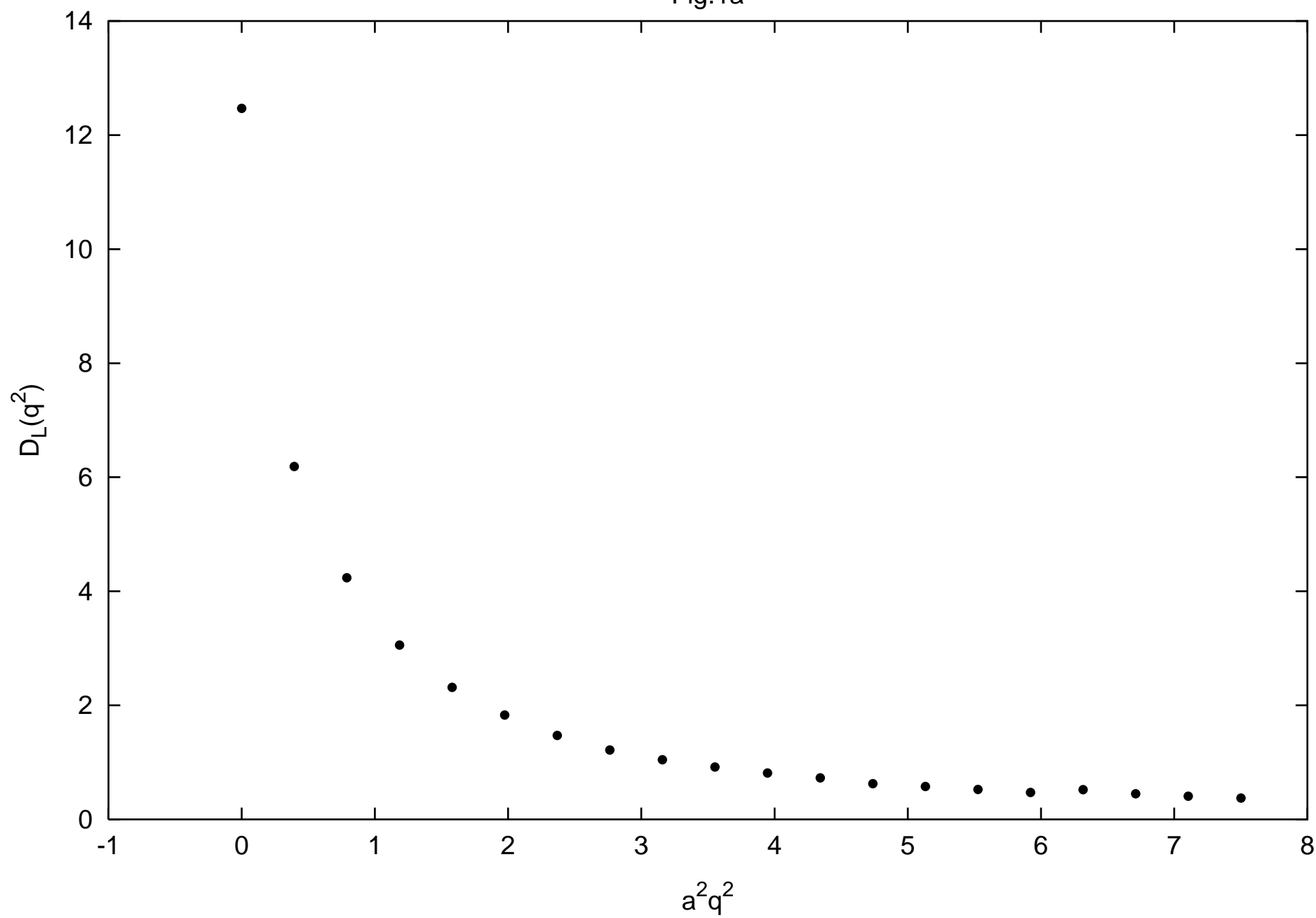


Fig.1b

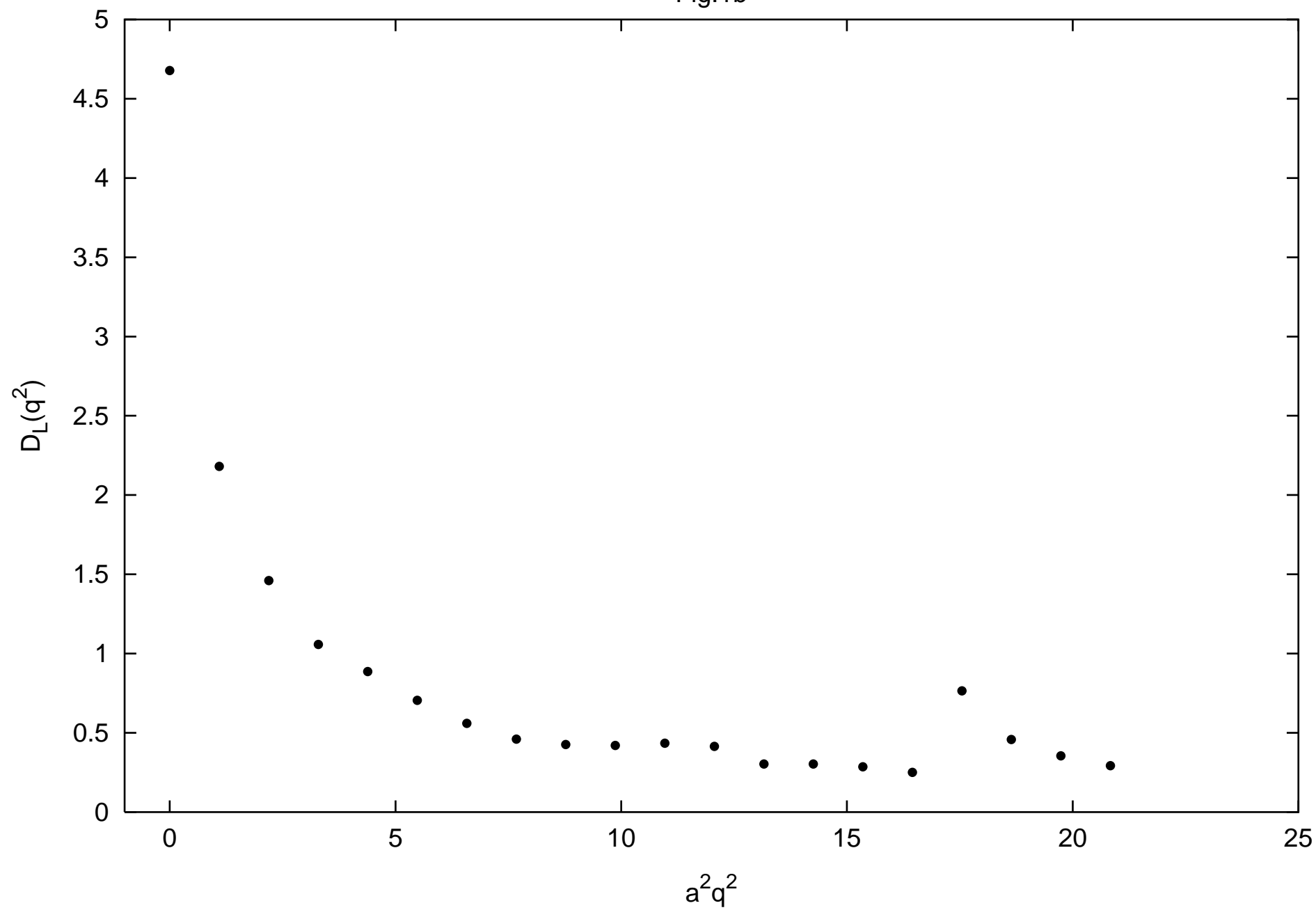




Fig.1c

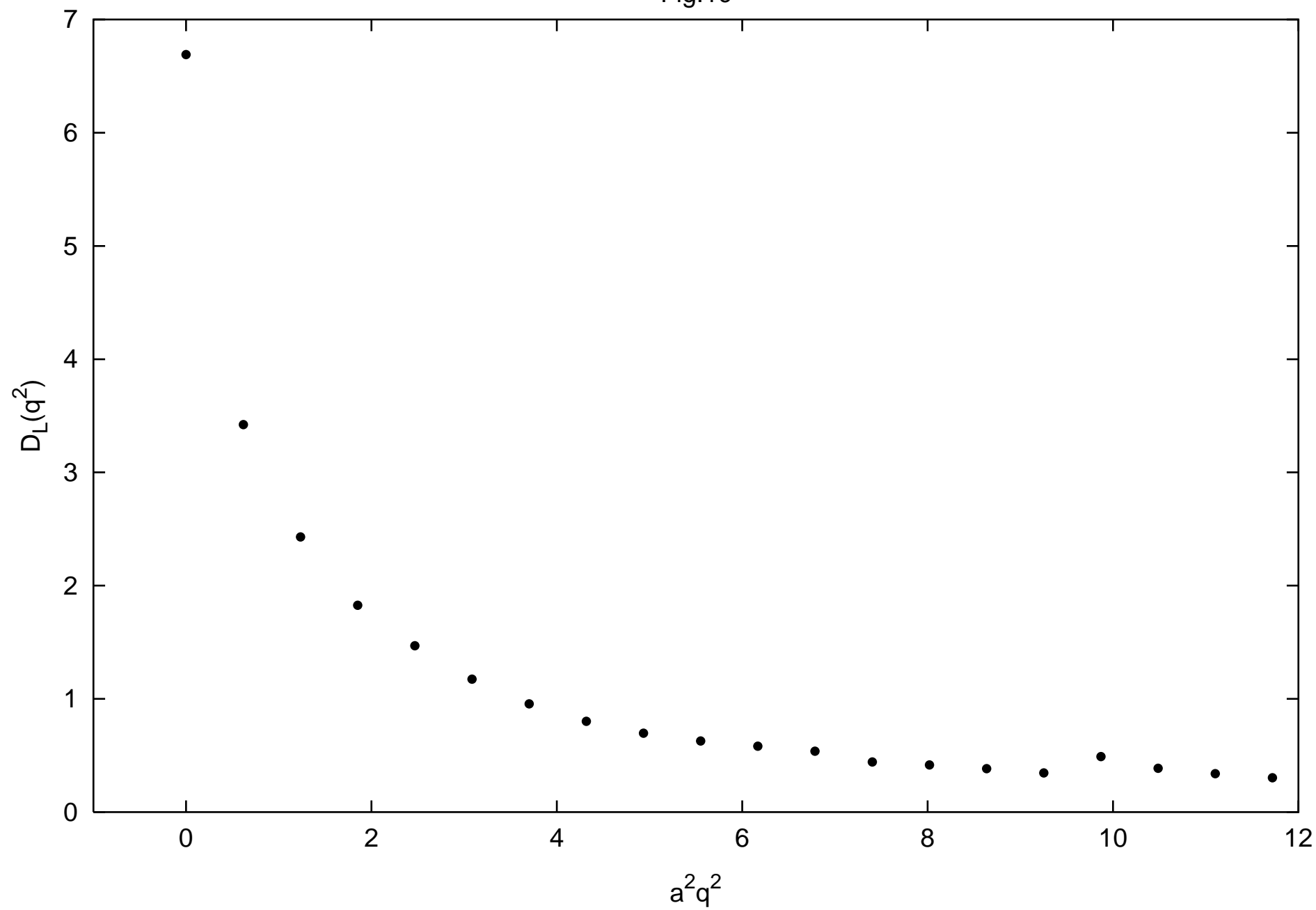


Fig.2a

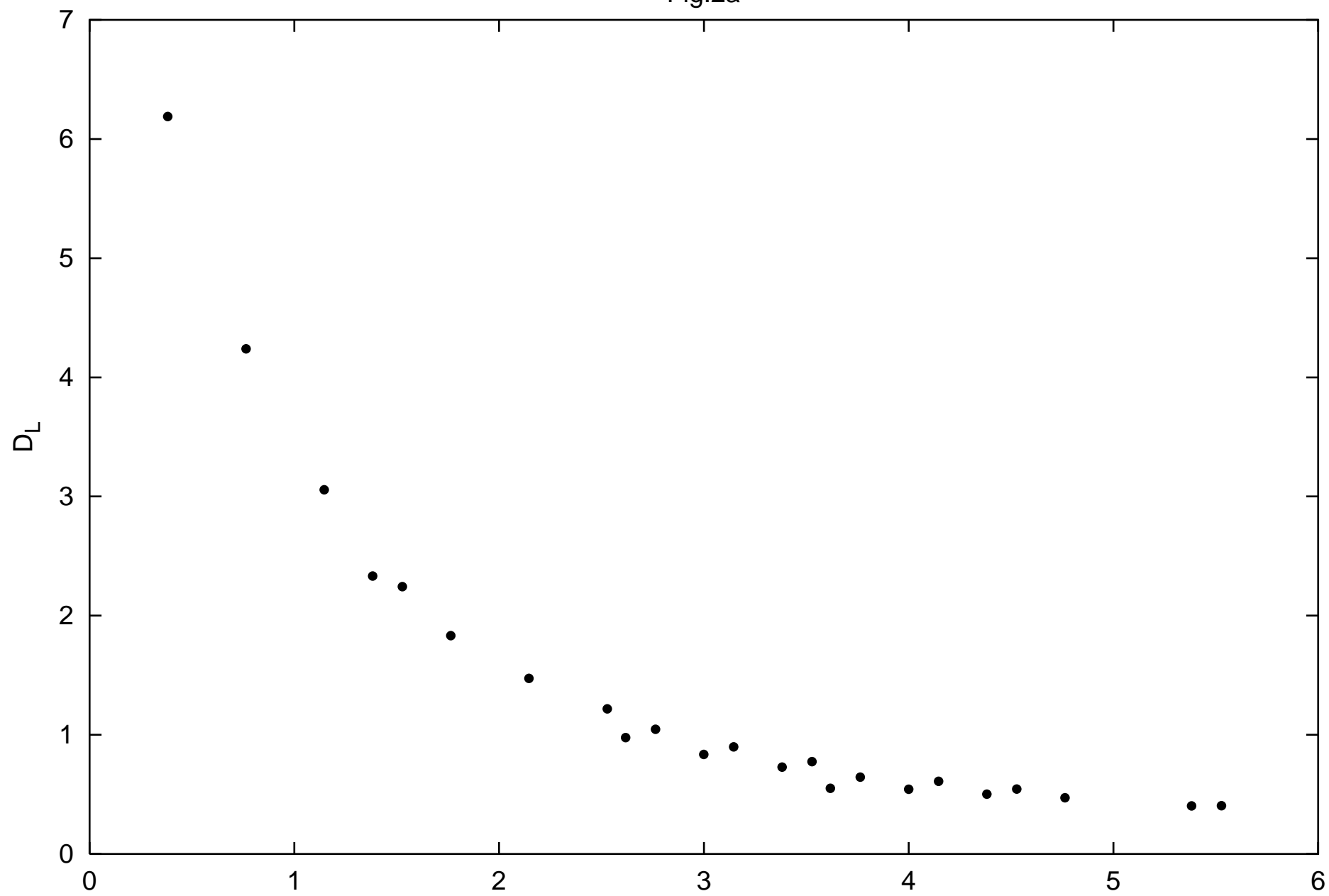


Fig.2b

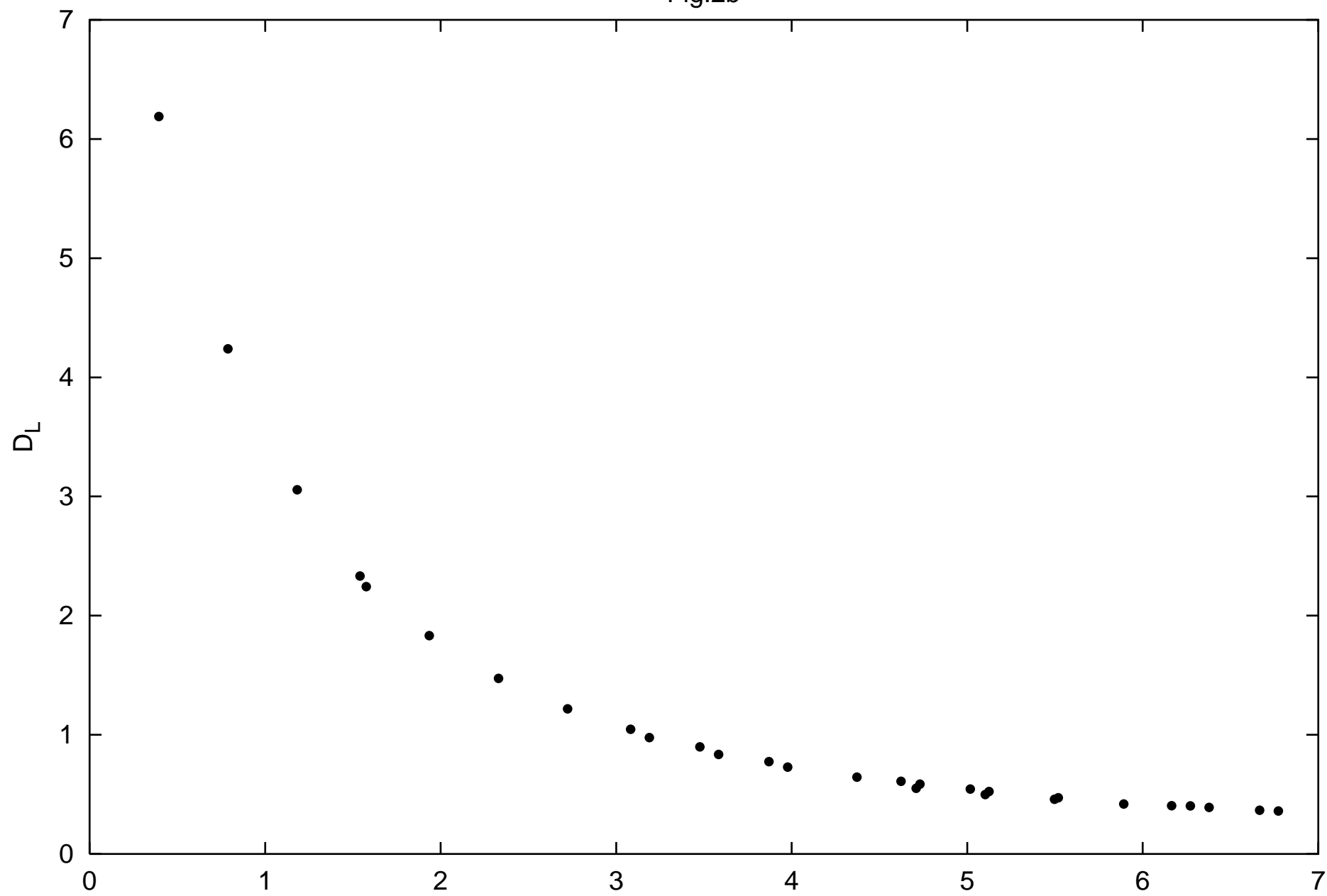


Fig.3a

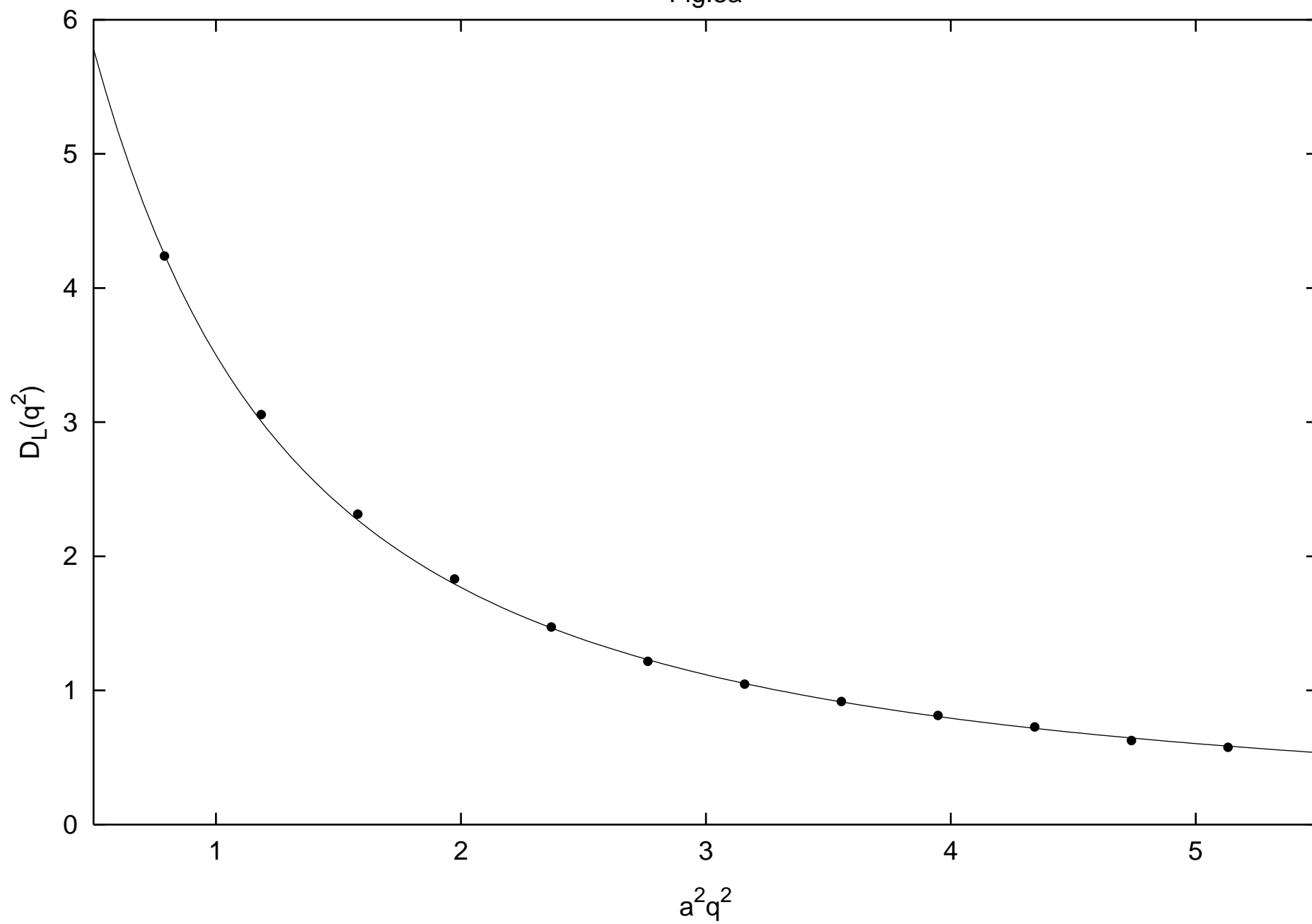


Fig.3b

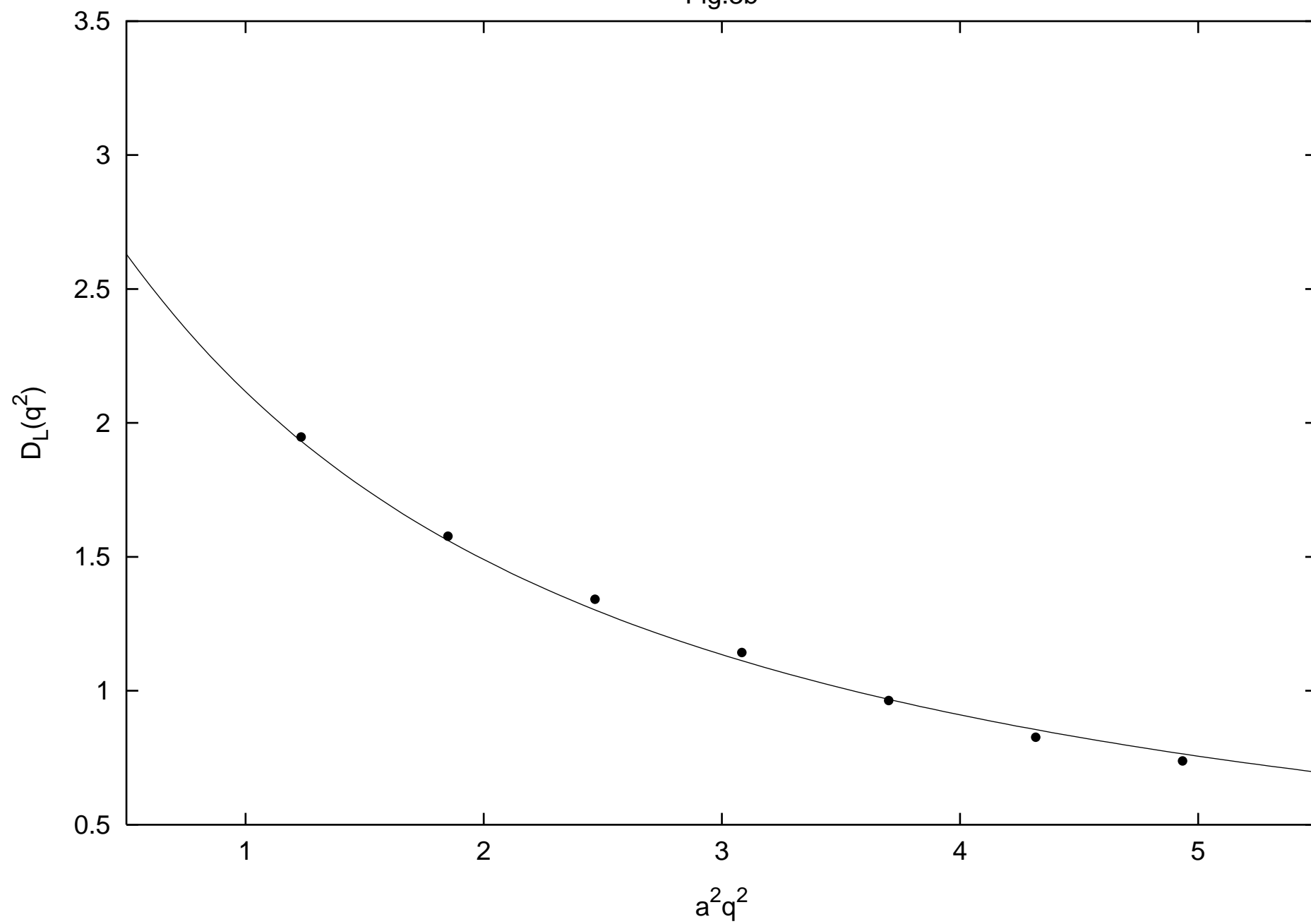


Fig.4

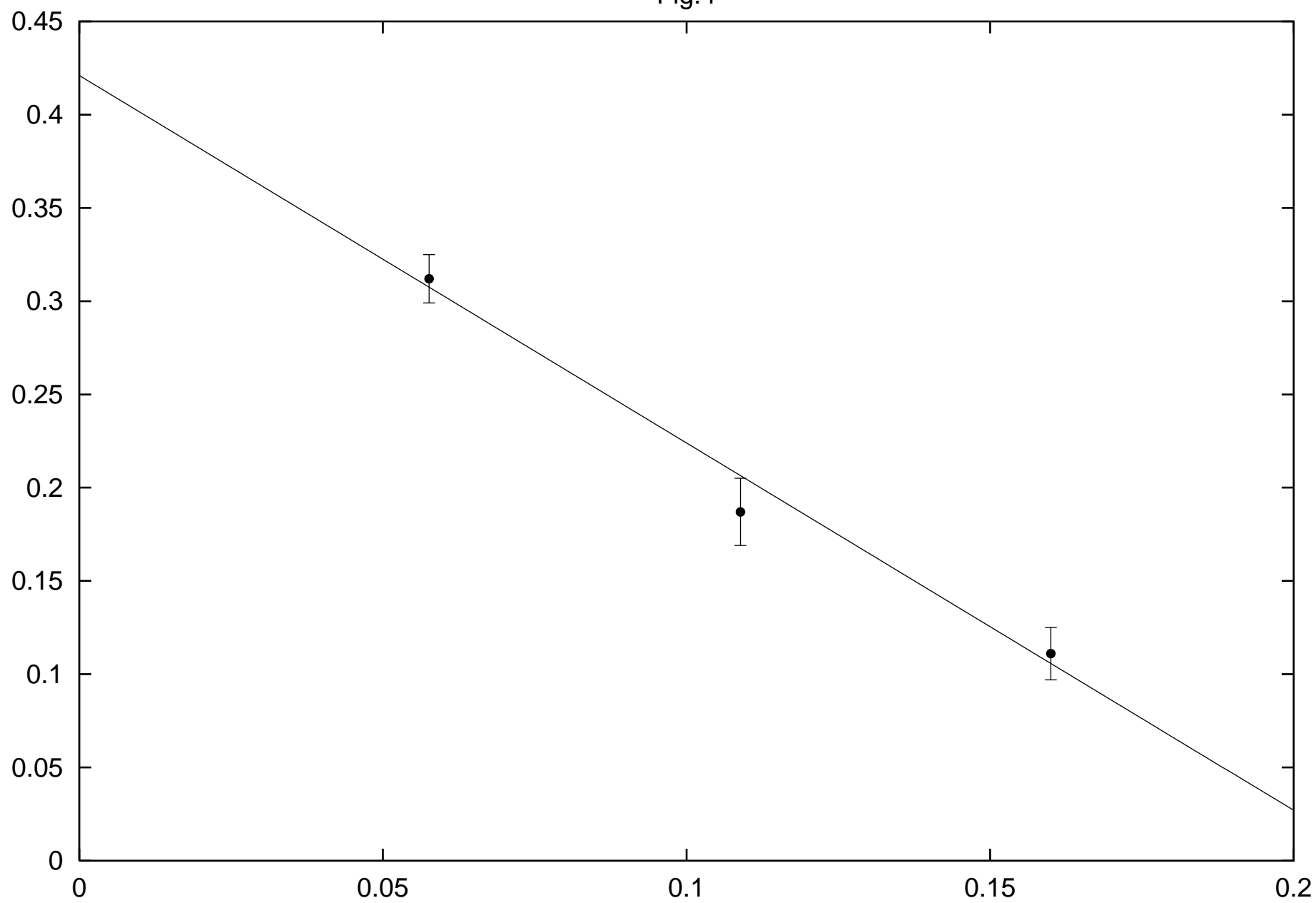


Fig.5a

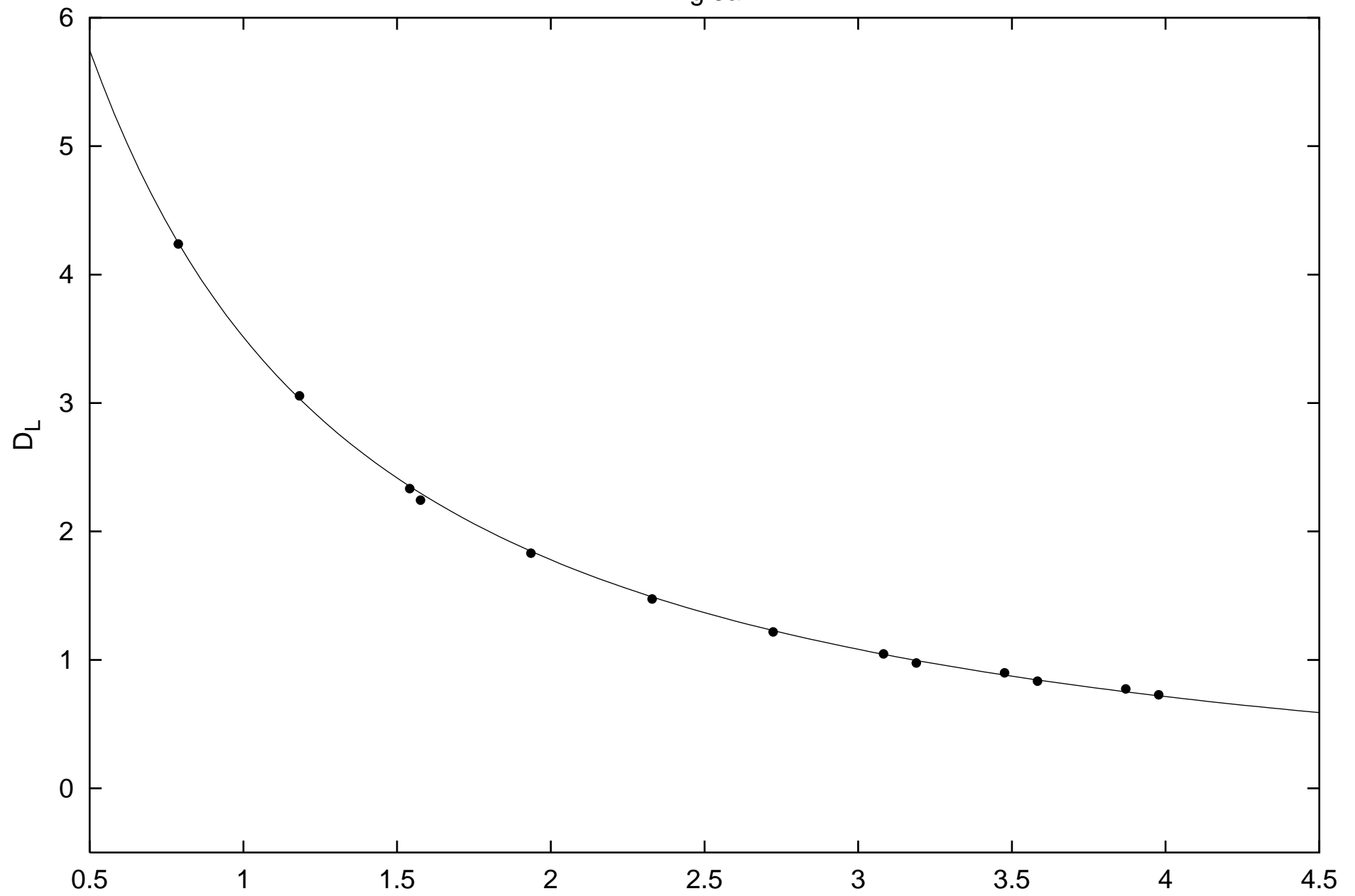


Fig.5b

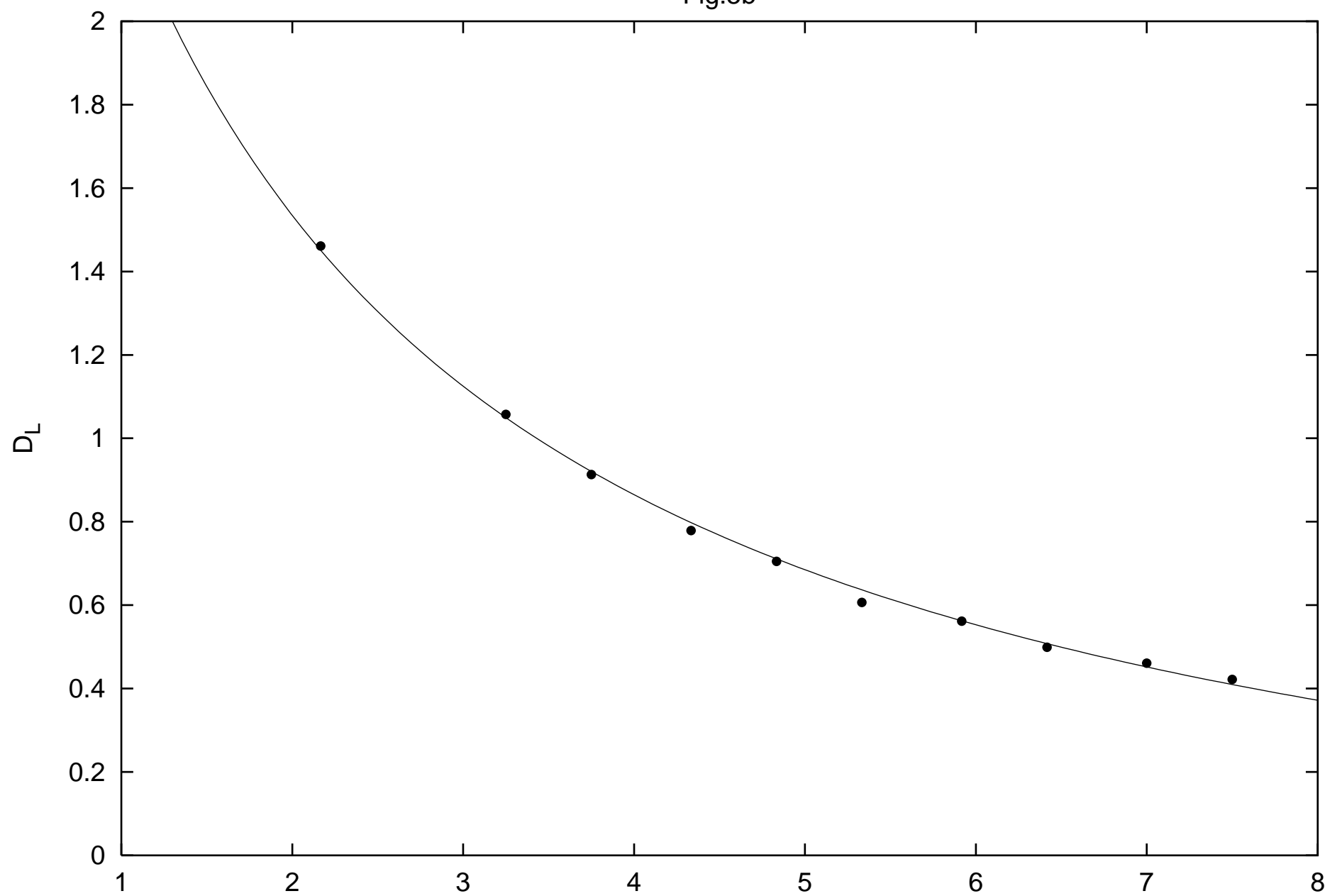




Fig.6

